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Research Article

Sequential Divestiture and Firm Asymmetry

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Simple Cournot models of divestiture tend to generate incentives to divest which are too strong, predicting that firms will break up into an infinite number of divisions resulting in perfect competition. This paper shows that if the order of divestitures is endogenized, firms will always choose sequential, and hence very limited, divestitures. Divestitures favor the larger firm and the follower in a sequential game. Divestitures in which the larger firm is the follower generate greater industry profit and social welfare, but a smaller consumer surplus.

1. Introduction

Firms often spin off divisions that compete directly with the parent business. For example, Siemens planned in early 2009 to divest its 34% stake in AREVA NP, a Franco-German joint venture in nuclear reactors, and develop its own nuclear capabilities (*The Economist*, 1/29/09). Fast-food chain Wendy's spun off fast-growing Tim Hortons in 2006 after realizing that "Tim's was beginning to compete directly with Wendy's" (*The Economist*, 9/23/06). Note that such spinoffs promise to increase competition rather than reduce it. On August 3, 2010, Ford Motor Company completed the sale of Volvo Cars to Chinese carmaker Geely for $1.8 billion. Ford CEO Alan Mulally said that the divestiture "will allow us to sharpen our focus on the Ford brand around the world and continue to deliver on our One Ford plan serving our customers with the very best cars and trucks in the world" (*AutoWeek*, 8/3/10). Geely's president Li Shufu remarked that Volvo would now have the freedom to "enter market segments that were previously closed to it because they were occupied by models from Jaguar, Land Rover or Ford itself" (*The Economist*, 3/31/10).

Separated from Ford, Volvo will surely compete with its old stablemate in the premium car market. Obviously Ford has created a competitor through the divestiture, and one wonders why it wanted to do so. Would not it have been better for Ford to keep Volvo under its roof and thus contain the competition between the two brands?[1] As these examples show, divestitures are very common in the business world, and many divestitures create direct competitors for the parent business.[2] This seems contrary to the common understanding that a company should seek to minimize competition. Why would a company ever want to spin off a business which will compete directly with itself? The answer lies in the responses from competitors. Although the newly created competition will eat into the parent company's business, it also applies pressure on rival companies. If the separation succeeds in squeezing existing rivals' market shares, the joint profit of the parent and the divested unit may increase, and the parent gains by extracting surplus through the unit's sale price. In other words, a company uses internal competition to gain competitive advantage over its external rivals. It is therefore not surprising that firms have an incentive to divest. What is surprising is that the incentive can be very strong. Standard Cournot models would predict that when firms produce homogeneous products at constant marginal cost and divestitures cost nothing, firms will divest into an infinite number of offsprings, leading to perfect competition [1].

This seemingly unreasonable prediction, that divestiture should always be profitable, can be called the divestiture paradox. Some solutions to the paradox have been offered: divestitures will be limited if they are costly [2] or products are differentiated [3, 4]. Those solutions assume that firms divest simultaneously. In real life, however, divestitures take time to formulate, and the decisions are often reported long before completion. It is therefore more appropriate to model divestitures as sequential choices rather than simultaneous ones. At the very least, the order of divesting should be endogenously determined.

This study investigated the incentive to divest when the order of divestiture is decided endogenously. In an industry with two firms producing homogeneous products, firms were found always to divest sequentially and doing so greatly limits the number of divisions. The divestiture paradox can therefore be solved by endogenizing the order of divestitures. Simultaneous divestiture is no longer an equilibrium because a firm is always better off playing a sequential game, either as a leader or as a follower. A leader can always choose its equilibrium strategy in the simultaneous game and achieve its payoff there, so it must be no worse off when it chooses differently in the sequential game. Because divestitures are strategic complements and the leader will be hurt by the follower's divestiture, the leader limits its own divestiture in order to constrain the follower's. As a result, a follower is also better off.

This research reflects the simple idea that the order of business choices should be endogenized whenever it is appropriate, especially when the results depend greatly on the order. Some choices such as prices or output levels may be regarded as being made simultaneously because they can easily be changed or are unlikely to be known to rivals beforehand, but divestitures take time and are usually reported by the media. Endogenizing the order of divestitures is particularly appropriate and easy because both firms benefit by moving from a simultaneous game to a sequential one, and such a move does not require any coordination. Even if originally the decisions were supposed to be simultaneous, a firm can easily turn them into a sequential one by, say, committing to a choice before other firms have made their decisions.

So the major novelty of this study is the endogenization of the order of divestitures. Another innovation is its way of modeling divestitures. Capital was assumed to be required in the production, which implies increasing marginal costs (in the short run). Compared with the commonly used formulation assuming constant marginal costs, this cost structure leads to a more reasonable modeling of divestiture: a divestiture decomposes the parent's capital so that each offspring is smaller than the parent. It also admits constant marginal cost as a special case where endogenizing the order of divestitures generates the most striking difference. With constant marginal costs, simultaneous divestitures induce each firm to divest into an infinite number of divisions, leading to perfect competition. When the order is endogenized, by contrast, the leader will not divest at all while the follower divests into only two divisions.

The presence of capital also enables the study of firm asymmetry. Divestitures can be shown to favor the larger firm and the follower. Divestitures increase social welfare at the expense of the industry's total profit. The leader is always hurt, but the follower may benefit if it is sufficiently large. A sequential divestiture in which the larger firm is the follower generates greater industry profit and social welfare, but a smaller consumer surplus than the alternative sequence. This is because the smaller firm tends to divest more than the larger one, and the follower tends to divest more than the leader. If the larger firm is the follower, the industry's overall divestiture is more limited and the resulting divisions are more balanced in size. The first effect helps industry profit and hurts consumer surplus, while the second effect improves production efficiency and hence helps industry profit and social welfare.

There is a small body of literature on divestiture (also known as strategic divisionalization), all assuming symmetric firms, constant marginal costs and simultaneous divestiture. Corchon [1] and Polasky [5] have demonstrated how a firm may gain competitive advantage by breaking itself up into autonomous units,[3] and found that the incentive to divest can be too strong, to the detriment of the divesting firms. Baye et al. [2] have suggested that divestitures will be limited if they are costly. All three of these studies assumed homogeneous products. By contrast, Ziss [3] and Yuan [4] showed that the incentive to divest can also be reduced by product differentiation; that is, the number of divisions decreases when products are more differentiated.[4] As a result, the divestiture paradox can be solved by introducing product differentiation: each firm will divest into a finite number of divisions when products are sufficiently differentiated, and perfect competition will not result.[5]

This paper assumes homogeneous products with increasing marginal costs. Such a setting is mathematically equivalent to assuming differentiated products with constant marginal costs [6]. It is therefore no wonder that both approaches can solve the divestiture paradox. Nevertheless, there are some subtle differences between the two formulations, as will be explained. More importantly, this paper's conclusions are based on the endogenized order of divestitures, while Ziss [3] and Yuan [4] both assumed simultaneous divestitures.

The incentive to divest echoes that for creating competing divisions within a company.[6] Creane and Davidson [7] have provided many examples of multidivisional firms that encourage internal competition, especially in the hotel, brewery, fast-food, automobile, and tobacco industries. They pointed out (p. 954) that "many firms offer multiple products that appear to be either identical or extremely close substitutes for one another." Conlin [8] found evidence of strong competition between different brands within a hotel chain. Kalnins and Lafontaine [9] discovered empirical regularities in the ownership of newly opened franchised units in the hotel industry and speculated that the regularities are best explained by an optimal balance between internal and external competition.

The divestiture paradox mirrors the better-known merger paradox, which says that firms tend to have too weak an incentive to merge [10]. The two paradoxes are actually two sides of the same coin—both are caused by a too strong response from competitors.[7] This is hardly surprising, as divestitures are simply reverse mergers. Endogenizing merger decisions, however, may or may not solve the merger paradox.[8] A recent study by Qiu and Zhou [11] treated divestitures and mergers in a single model, which solves both paradoxes simultaneously, as the interaction between the two restructuring activities weakens the incentive to divest and strengthens the incentive to merge. Reflecting in a merger context the logic of using internal competition to gain competitive advantage, Mialon [12] demonstrated that two merging firms may choose to remain as independent and competing divisions after proper reallocation of capital between them.[9]

A number of studies have addressed the order of moves in two-player games. Dowrick [13] endogenized the order by allowing duopolists to simultaneously choose a role as either the leader or the follower before they engage in competition in the product market. Hamilton and Slutsky [14] have suggested that two perfectly symmetric firms may choose to play a sequential game, and the order of moves can then be endogenized through either observable delay or action commitment. Finally, Henkel [15] has suggested that a player may choose a role somewhere between a leader and a follower by making a commitment to a role which can be revoked later at some cost.

2. The Model

Assume that two firms, indexed by x and y, produce a homogenous good. The production cost of firm i ($i \in \{x, y\}$) is assumed to be $C(t_i, q_i) = q_i^2/2t_i$, where t_i is i's capital stock and q_i is its output. This cost function has been used previously in merger studies [16–18] and can be viewed as a short-run cost derived from a Cobb-Douglas production function.[10]

The two firms play a three-stage game. In stage one (the timing stage), the two firms determine the sequence of their future divestitures by simultaneously choosing a role r_i from $\{L, F\}$, where L means leader and F means follower. If $r_x \neq r_y$ the two firms will divest sequentially in stage two, with the one choosing L divesting first and the other divesting afterward. If $r_x = r_y$, they will divest simultaneously. In stage two (the divestiture stage), the two firms divest according to the sequence determined in the previous stage.[11] To divest is to break up a firm's capital into several smaller units, termed divisions, which will compete independently in the product market. Each parent firm's payoff will be the sum of all its divisions' future profits. Since it is optimal for a parent to distribute its capital equally among its divisions,[12] the divestiture decision comes down to a single choice of the number of divisions, which for simplicity will be treated as a continuous variable. In stage three (the competition stage), all the divisions from both firms compete independently and simultaneously à la Cournot. The product demand is $p = a - bQ$, where $a, b > 0$ and Q is the total output of all competing divisions.

3. Analysis

3.1. Stage Three: Cournot Competition. The appendix shows how to derive the Cournot outcome. Denote the set of all competing divisions by D. For division $k \in D$, let $g_k \equiv bt_k/(1 + bt_k)$ and $G \equiv \sum_{d\in D} g_d$. Division k's Cournot profit is then $\pi_k = (a^2 g_k(1 + g_k))/(2b(1 + G)^2)$. Without loss of generality,[13] normalize $a \equiv \sqrt{2}$ and $b \equiv 1$. Then

$$\pi_k = \frac{g_k(1 + g_k)}{(1 + G)^2}, \quad (1)$$

where

$$g_k = \frac{t_k}{1 + t_k}, \quad G = \sum_{d\in D} g_d, \quad (2)$$

with $q_k = (a/b)(g_k/(1 + G))$, $Q = (a/b)(G/(1 + G))$, and $p = a/(1 + G)$. Division k's marginal cost at equilibrium is therefore $mc_k = q_k/t_k = (a/(b(1 + G)))(1/(1 + t_k))$, and its market share is $s_k \equiv q_k/Q = g_k/G$.

3.2. Stage Two: Divestitures. Let $i \in \{x, y\}$ and $j = \{x, y\} \setminus i$. If firm i chooses to form n_i divisions, each division's capital will be t_i/n_i. Let $g_i = (t_i/n_i)/(1 + (t_i/n_i)) = t_i/(n_i + t_i)$. Firm i's payoff is then

$$\pi_i(n_i, n_j) = \frac{n_i g_i(1 + g_i)}{(1 + n_i g_i + n_j g_j)^2}. \quad (3)$$

Given n_j, i's optimal choice is derived as $n_i^* \equiv \arg\max_{n_i>0} \pi_i(n_i, n_j) = 1 + n_j g_j$. Define a firm's best response to its rival's choice of n_j as

$$n^*(n_j) \equiv 1 + n_j g_j = 1 + \frac{n_j t_j}{n_j + t_j}. \quad (4)$$

If the two firms divest simultaneously, the equilibrium is determined by solving the two best response equations, $n_i^M = n^*(n_j^M)$ and $n_j^M = n^*(n_i^M)$, where the superscript M indicates simultaneous divestiture. As a result,

$$n_i^M = \frac{1 + 2t_j + \sqrt{(1 + 2t_i)(1 + 2t_j)(1 + 2t_i + 2t_j)}}{2(1 + t_i + t_j)}. \quad (5)$$

If the two firms divest sequentially, the follower will adopt the best response: $n^F = n^*(n^L)$. Anticipating this, the leader's optimal choice is $n^L = \arg\max_{n_i>0} \pi_i(n_i, n^*(n_i))$, which has a unique solution on $(0, \infty)$ defined by

$$\beta(n^L) = 0, \quad (6)$$

where

$$\begin{aligned}\beta(n^L) \equiv &-\left[(1 + t_L + t_F)^2 + t_F^2\right](n^L)^3 \\ &+ \left[(1 + t_F)(1 + 2t_F - t_L^2) - 4t_L t_F^2\right](n^L)^2 \\ &+ t_L\left[(1 + 2t_F)(2 + 2t_F + t_L) - 2t_L t_F^2\right] n^L \\ &+ t_L^2(1 + t_F)(1 + 2t_F).\end{aligned} \quad (7)$$

3.3. Stage One: The Order of Moving. Denote firm i's equilibrium choice of the number of divisions by n_i^L when i is the leader in a sequential divestiture (and accordingly j is the follower), by n_i^F when i is the follower (and j is the leader), and by n_i^M when i and j divest simultaneously. The corresponding equilibrium payoff for i is denoted by π_i^R, where $R = L, F, M$. The appendix proves the following ranking, which is central to the major conclusions of this analysis.

Lemma 1. $n_i^L < n_i^F < n_i^M$ *and* $\pi_i^F > \pi_i^L > \pi_i^M$.

Because $\pi_i^F > \pi_i^L > \pi_i^M$ and $\pi_j^F > \pi_j^L > \pi_j^M$, it is never an equilibrium for the two firms to choose $r_i = r_j$ in stage one, and it is always an equilibrium for $r_i = L$ and $r_j = F$ or for $r_i = F$ and $r_j = L$. Therefore, consider the following.

Proposition 2. *(i) Simultaneous divestiture is never an equilibrium.*

(ii) Sequential divestiture (with either firm serving as the leader) is always an equilibrium.

(iii) In equilibrium, n_i is finite and $\pi_i > 0$ even though the products are homogeneous and divestiture is costless.

Propositions 2(i) and (ii) are corollaries of Lemma 1. Proposition 2(iii) is true because $n_i^L < n_i^F < n_i^M$ (Lemma 1) and n_i^M is finite.

4. Discussion

Proposition 2 delivers the key message of the analysis which is that divestitures will be limited in scope if firms can choose the order of their divestitures. The following discussion presents the intuition suggesting this result, the unique features of the model, and the properties of the equilibrium. To understand the discussion, it is useful to first look at the incentives for and effects of a unilateral divestiture.

4.1. The Incentive to Divest. In what follows, the number of divisions will be called the scope of divestiture, or simply divestiture. Increasing a firm's divestiture generates the following tradeoff:[14] it increases the competition between the firm's own divisions, hurting the firm, but the increased competition also forces rival firms to retreat, benefiting the firm. When divisions from the same parent compete independently, they create a negative externality for one another. From the viewpoint of the parent firm, fixing its Cournot rivals' outputs, these divisions produce too much. Rivals' outputs, however, will not be fixed. Precisely because the divested divisions overproduce, rivals will produce less (as outputs are strategic substitutes), which benefits the divesting firm. The tradeoff between the two effects determines the optimal scope of a divestiture.

Now consider the effects of increasing one firm's divestiture while holding the other's constant. Firm i's profit has been given by (3): $\pi_i = (n_i g_i(1 + g_i))/(1 + n_i g_i + n_j g_j)^2$. The joint output of all its divisions is $q_i = (a/b)\,(n_i g_i/(1 + n_i g_i + n_j g_j))$, and the total market share of those divisions, which for simplicity is called i's market share, is

$$s_i \equiv \frac{q_i}{q_i + q_j} = \frac{n_i g_i}{n_i g_i + n_j g_j}. \tag{8}$$

All these variables depend on the comparison between $n_i g_i$ and $n_j g_j$. Let us call $n_i g_i \equiv n_i t_i/(n_i + t_i)$ firm i's *dispersion*, which obviously increases with the scope of its divestiture. For a fixed n_j, therefore, increasing n_i increases i's dispersion and thus the industry's aggregate dispersion, while j's dispersion remains unchanged. As a result, increasing a firm's divestiture will expand the total output and market share of its divisions and depress the joint output, market share, and profit of its rival's divisions.

Direct observation of the best response (4) reveals the following properties:

$$\frac{\partial n^*}{\partial t_j} > 0, \qquad \frac{\partial n^*}{\partial n_j} > 0. \tag{9}$$

That is, a firm will divest more if its rival is larger or divests more. The second property indicates that divestitures are strategic complements. Both properties arise for the same reason. When the rival increases its capital or divestiture, its dispersion increases, expanding its market share and reducing the market share of the firm contemplating divestiture. As has been discussed earlier, the optimal divestiture is determined by the tradeoff between the competition among the divesting firm's own divisions and the retreat of the rival firm's divisions. When the rival's market share is larger, the advantage of divestiture increases because there are now more rival divisions to respond to the divestiture, while the disadvantage of the divestiture will be smaller because the divesting firm's divisions are not earning much anyway. As a result, the firm will divest more.[15]

4.2. Endogenized Order and the Scope of Divestiture. The previous discussion concerns the incentives for and effects of a unilateral divestiture. When both firms divest, the equilibrium is governed by two forces: a firm's divestiture hurts rival firms and divestitures are strategic complements. The two forces imply that the two firms overdivest to each other's detriment. In fact, the incentive to divest can be so strong that (assuming a homogeneous product, linear demand, constant marginal costs, and costless divestitures) firms will divest into an infinite number of divisions even when the industry has only two firms, leading to perfect competition [1].

This seemingly unreasonable prediction, that breaking up a firm is always profitable, is the "divestiture paradox." Both this and the merger paradox are driven by the same force: rival firms are too responsive to other firms' restructuring, be it a divestiture or a merger. One solution to the divestiture paradox is to make divestitures costly [2]. A second solution is to let firms produce differentiated products so that the response from rival products is weakened [3, 4].

Surprisingly, all previous studies have assumed that firms divest simultaneously. According to Lemma 1, however, both firms will be better off playing a sequential game rather than the simultaneous one. The reason why sequential moves are better for both players can be understood from the two forces at work. The leader in a sequential game will choose a smaller scope of divestiture than in the simultaneous game, because doing so will constrain the follower's divestiture and hence its damage to the leader. Since the follower's payoff increases when its rival divests less, the follower is better off. The leader must also be better off because it can guarantee itself the simultaneous-move payoff by choosing its simultaneous-game strategy in the sequential game and can only do better if it chooses differently.

Once it has been established that both firms are better off in a sequential game, it is hard to see why they would play the simultaneous one which gives them an inferior outcome. Scholars have never explained why they have assumed firms divest simultaneously, but as has been shown, once the order of moves is endogenized, divestitures will be carried out sequentially. In real life, the order of business decisions is usually determined endogenously. Endogenization is particularly easy in the case of divestiture because both firms benefit by moving from the simultaneous game to a sequential game, and the move does not require any coordination, which is not the case in many other situations (such as the prisoners' dilemma) where the players are trapped in an inferior outcome.

To summarize, even if products are homogeneous and divestitures are costless, firms will not divest into an infinite number of divisions and therefore will not face perfect competition. Proposition 2 demonstrates that, after all, firms will find a way to constrain mutually damaging divestitures. The divestiture paradox disappears as soon as the order of divestitures is endogenized.

4.3. Cost Structure. Having established the major conclusion of the paper, we are now ready to discuss the features of the model and the properties of the equilibrium. In addition to an endogenized order of divestiture, this model has assumed the involvement of capital, which leads to a marginal cost (*mc*) that increases with output. By contrast, all previous models have assumed constant *mc*. Increasing marginal cost yields implications for divestiture which are more reasonable. If marginal costs are constant, each extra division will be an exact replica of the parent firm: to divest amounts to create something out of nothing. Divestitures modelled in this way contradict the general understanding that a division should be somewhat smaller than the parent firm. By contrast, when marginal costs are increasing, each division gets a portion of the parent's capital and is therefore smaller (in the sense of having higher production costs) than the parent.[16]

Note that modeling divestitures when breaking up a firm is reasonable and meaningful only when *mc* is increasing. To understand this, it is helpful to consider divestitures as reverse mergers. In merger, the merged entity takes over the production facilities of the merging firms and optimizes its production using those facilities. It can be easily shown that such within-firm optimization will give rise to a cost function of the form $q^2/(2(t_i+t_j))$ for the merged firm, where t_i and t_j are the capital stocks of the two merging firms. Because the merged firm's cost function is that of a single firm with capital $t_i + t_j$, we may regard the merger as a process of pooling the merging firms' capital. As a reverse merger, then, a divestiture can be viewed as decomposing the parent's capital. That explains the modeling of divestiture in this study and why a division's cost is higher than the parent's.

To summarize, the cost structure assumed in this study differs from that assumed in previous studies in two respects: marginal cost is increasing rather than being constant, and a division is smaller than the parent. The second aspect is to generate a reasonable modeling of divestitures. To justify such modeling, however, we need to introduce capital stock, which naturally gives rise to the first aspect of increasing *mc*. So increasing *mc* is a necessary element of the model to justify why and how a division is smaller than the parent and is therefore essential for modeling divestitures.

Once capital is introduced into the model, the formulation with constant *mc* becomes a special case where both t_i and t_j approach infinity so that the two firms' marginal costs approach zero. In that case, a division is indeed identical to the parent.

At this point, it is helpful to discuss the relationship between increasing marginal cost and product differentiation. As Vives [6] has pointed out, assuming homogeneous products with increasing *mc* is mathematically equivalent to assuming differentiated products with constant *mc*. Economically the two situations produce the same effect of weakening the interaction between Cournot competitors. That is why either setting can solve the divestiture paradox: even with constant *mc*, the scope of divestitures will be limited if products are differentiated [3, 4]. Although mathematically equivalent, assuming increasing *mc* has the advantage of avoiding the complication of dealing with different degrees of substitution for products within and across groups.[17] Finally, increasing *mc* and product differentiation can be combined in the modeling, but that would not change any of the qualitative results.[18]

4.4. The Role of Capital. This model differs from those previously proposed both in its cost structure and in its endogenized order of moves. It must be pointed out that the assumption of increasing *mc* is not driving the results of Proposition 2. Even when marginal costs are constant, the scope of divestiture is still limited as long as the order of divestiture is determined endogenously. Inspecting the simultaneous and sequential divestiture choices depicted in (5) and (6), we have the following lemma.

Lemma 3. *If* $t_i \to \infty$ *and* $t_j \to \infty$, *then* $n^L = 1$ *and* $n^F = 2$, *but* $n_i^M = \infty$.

The lemma says that even when marginal costs are constant, divestiture is limited if the firms divest sequentially. In particular, the leader will not divest at all while the follower divests into two divisions. This is in sharp contrast with the result when firms have to divest simultaneously. In that case, each firm will divest into an infinite number of divisions, leading to perfect competition, as previous studies have concluded [1]. The reason for such contrast is that, as has been explained in Section 4.2, when divestitures are sequential, the leader has a chance to influence the follower's choice, and it will limit its own divestiture in order to constrain the follower's. Such reasoning is independent of the cost structure and is therefore still valid when marginal costs are constant. Proposition 2 still holds, so the equilibrium will be sequential divestitures of limited scope.

To further investigate the difference between sequential and simultaneous divestitures and how the difference depends on capital, consider the overall degree of divestiture in the industry and the two firms' profits. Because the firms are asymmetric in terms of both their capital stocks and the order of divestiture, adding up the number of the two firms' divisions is meaningless. The scope of divestiture industrywide is better measured by the Cournot equilibrium price $p = a/(1+G)$. To visualize the effects, assume symmetric firms ($t_i = t_j = t$) and use the no-divestiture situation ($n_i = n_j = 1$) as the benchmark.[19] Superscripts Q, M, and 0 indicate sequential, simultaneous, and no divestiture. Figure 1 shows the price and profits as functions of t. Since these two variables depend on t even without divestiture, the effect of divestiture is captured in the price ratios (the left panel of Figure 1) and profit ratios (the right panel).

A firm's capital stock represents its size, more specifically its capacity, relative to the demand. Capital matters for divestiture because it determines how responsive a firm is to other firms' output changes. When both firms are very small, each operates with a very steep marginal cost curve. Since there is little interaction between the two firms, there is no need to divest: When $t \rightarrow 0$, $n^L = n^F = n^M = 1$, so the order of divestiture does not matter. As both firms' capital increases, the interaction between them becomes stronger, and so does the incentive to divest. Equilibrium divestiture increases regardless of the order of moves, as p^Q/p^0 and p^M/p^0 both decline with t. However, the overall divestiture is always larger with simultaneous moves than with sequential moves ($p^M < p^Q$), and capital matters more in simultaneous divestiture than in sequential divestiture. At the limit, when t approaches infinity, $p^M/p^0 = 0$ but $p^Q/p^0 = 3/4$, meaning that simultaneous divestiture leads to perfect competition, but sequential divestiture will lead to competition that is greatly constrained (the market price is 3/4 of the level when neither firm divests). In terms of profits, when mc is constant (t is infinity), simultaneous divestiture leads to zero profit for both firms, but sequential divestiture gives substantial profits to both firms, with the follower earning twice as much as the leader: $\pi_i^L/\pi_i^0 = 9/16$, while $\pi_i^F/\pi_i^0 = 9/8$. Note that the follower's profit is larger than without divestiture ($\pi_i^F/\pi_i^0 > 1$) when t is sufficiently large.

To summarize, for any given stock of capital, there is a difference between simultaneous moves and sequential moves, and the difference increases with the amount of capital, becoming most dramatic as capital approaches infinity, that is, when marginal costs are constant.

4.5. Firm Asymmetry. All previous models have assumed symmetric firms. Because the major innovation of this study is to endogenize the order of moving, it has for simplicity focused on an industry with only two firms. This simplification allows the firms to be asymmetric, which offers an opportunity to study how divestitures change the firms' relative strengths and market shares. Since the equilibrium choice depicted in (6) does not admit any closed-form solution, numerical calculation is sometimes required. Doing so involves little loss of rigor, as there are only two parameters (t_L and t_F) and all the results can be inspected using 3D graphs.[20]

Proposition 4. *In equilibrium (i.e., when firms divest sequentially), (i) $s_i^L < s_i^F$ and (ii) $\pi_i^L + \pi_j^F > \pi_j^L + \pi_i^F$ if and only if $t_i < t_j$.*

Proposition 4(i) says that a firm's market share as the follower is always greater than its share as the leader. This is consistent with Lemma 1 which says that a firm's profit as the follower is greater than its profit as the leader, and for the same reason. The leader has an extra incentive to constrain its expansion in order to restrict the follower's expansion, while the follower does not face such a constraint.

Proposition 4(ii) says that the industry's total profit is greater when the larger firm is the follower. There are two reasons for this result. First, the smaller firm tends to divest more than the larger one,[21] and the follower tends to divest more than the leader (Lemma 1). If the larger firm is the follower, the industrywide divestiture is more limited (see Proposition 6). The industry's output will be smaller, and the industry's total profit will be greater. Second, even with the same industry output, production cost is lower when the larger firm is the follower. Recall that a firm divests more when it is the follower (Lemma 1), so when the larger firm is the follower, the divided capital will be more balanced; therefore, the industry's total production cost will be lower.

As Proposition 2 shows, both sequences in sequential divestitures constitute equilibria. To arrive at a unique equilibrium, suppose that the two firms bid to be the follower in some formal game—a kind of war of attrition. In that case, the equilibrium sequence will be the one that yields the greater industry profit. According to Proposition 4(ii), then, the smaller firm will be the leader in such a unique equilibrium, while the larger firm will be the follower. Since the follower tends to divest more, this would make the size distribution among the divisions more balanced.

4.6. Welfare. The equilibrium price for the industry's homogeneous product has been shown to be $p = a/(1 + G)$. When the industry's aggregate dispersion G increases, the price drops, so the consumer surplus increases. Since $G \equiv n_i g_i + n_j g_j$ and firm i's dispersion, $n_i g_i \equiv n_i t_i/(n_i + t_i)$, increases with n_i, consumer surplus is improved if either firm's divestiture increases.

Social welfare differs from consumer surplus because production efficiency now matters. Efficient production requires all Cournot competitors' marginal costs to equate in equilibrium. Section 3.1 has shown that each division from firm i has an equilibrium marginal cost proportional to $1/(1 + (t_i/n_i))$, where t_i/n_i is an i-division's capital level. The industry's overall production efficiency will then be improved if t_i/n_i is closer to t_j/n_j, that is, if divestitures generate divisions that are more balanced in size. Conversely, production efficiency and hence social welfare may be damaged if divestitures increase the size discrepancy between the two firms' divisions. This would be the case if the smaller firm's divestiture increased while the larger firm's divestiture remained constant.[22] To see an example, note that social welfare is

$$W \equiv \mathrm{CS} + \pi_i + \pi_j = \frac{G}{1+G} + \left(\frac{G}{1+G}\right)^2 h, \tag{10}$$

where CS is consumer surplus and $h \equiv \sum_{d \in D} s_d^2 = (n_i g_i^2 + n_j g_j^2)/G^2$ is the Hirfindahl index. Let $t_i = 10$ and $t_j = 1$.

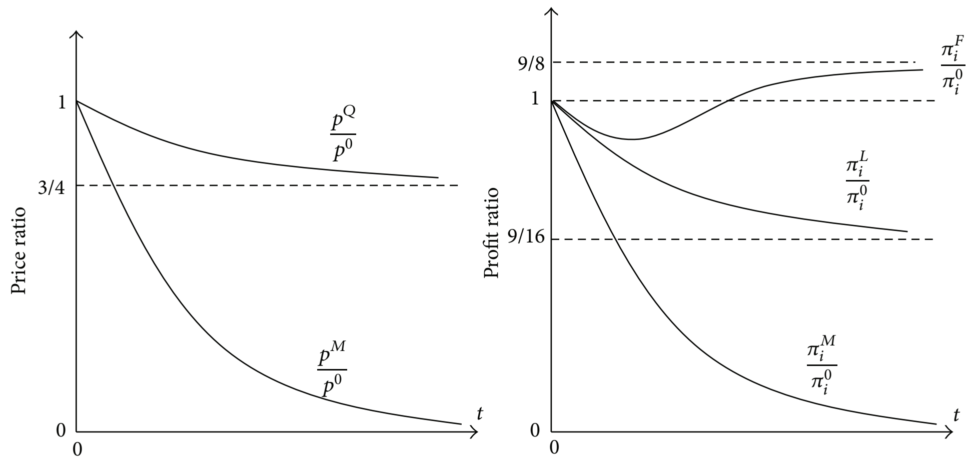

FIGURE 1: The comparison between sequential and simultaneous divestitures when the two firms are symmetric.

Holding $n_i = 1$, social welfare declines monotonically as n_j increases. Holding $n_j = 1$, welfare increases with n_i up to $n_i = 50$ and then declines. If n_i and n_j both increase while maintaining the balance between divisions, for example, by keeping $t_i/n_i = t_j/n_j$, then social welfare always increases. This discussion of welfare would apply where the number of divisions is chosen by a social planner or an antitrust authority.

Turning to equilibrium divestitures, we must first establish the following result.

Lemma 5. *$n^L > 1$ when $t_F > 0$, and $n^L = 1$ when $t_F = 0$.*

The lemma says that the leader in a sequential game will always divest to some extent ($n^L > 1$) unless it faces no competition (i.e., the follower has zero capital). Lemmas 1 and 5 then imply that $n^M > n^F > n^L > 1$. As a result, $G(n_i^M, n_j^M) > G(n_i^L, n_j^F) > G(1,1)$. That is, the consumer surplus will be increased by sequential divestitures and will be increased even more by simultaneous divestitures.

Between the two sequences of sequential divestitures, the one in which the larger firm is the follower generates the greater social welfare (and the greater total industry profit, as established earlier), but a smaller consumer surplus than the alternative sequence. The reason is that when the larger firm is the follower, the size of the divisions is more balanced, making production more efficient. This gain in production efficiency outweighs the loss of consumer surplus.

The above results can be summarized as follows.

Proposition 6. *(i) Unilaterally increasing the extent of a firm's divestiture always increases the resulting consumer surplus, but it may damage social welfare. Increasing the extent of both firms' divestitures while maintaining the balance between their divisions always improves social welfare.*

(ii) $CS^M > CS^Q > CS^0$; $W^M > W^0$, $W^Q > W^0$. It may happen that $W^M < W^Q$ if the smaller firm serves as the leader.

(iii) $CS^Q(L = i,\ F = j) > CS^Q(L = j,\ F = i)$ and $W^Q(L = i,\ F = j) < W^Q(L = j,\ F = i)$ if and only if $t_i > t_j$.

5. Concluding Remarks

This study has two distinctive features: first, the endogenized order of divestitures which solves the divestiture paradox. Second, capital is introduced into the production function so that marginal costs are increasing. The second feature, although not essential for resolving the divestiture paradox, is nonetheless desirable because it enables more reasonable modeling of divestitures and admits the commonly assumed constant marginal costs as a special case. A firm's capital stock is also a natural way to model its size and hence firm asymmetry. Combining the two features generates some new insights. For example, it reveals that the leader may divest less when the follower becomes bigger, and that the follower may benefit from divestitures. These conclusions are different from those applicable to simultaneous divestitures, where a firm always divests more when the rival is bigger and both firms are hurt by divestitures.

The model assumes linear demand and two firms, but linearity of demand is not driving the divestiture paradox. Corchon and Gonzalez-Maestre [19] have shown that Corchon's [1] conclusion still holds under mild conditions even if the demand is nonlinear. The insight of this paper should then still apply. The number of firms does not matter either. When there are three or four firms, the overall divestiture is still much smaller with sequential rather than simultaneous moves. The reason is the same as in the main model: because divestitures are strategic complements and because a firm suffers from other firms' divestitures, an early mover will reduce its own divestiture in order to constrain later movers' divestitures.

Appendix

Derivation of Cournot equilibrium. For division $k \in D$, its profit is $\pi_k = pq_k - C(t_k, q_k) = (a - bQ_{-k} - bq_k)q_k - (q_k^2/2t_k)$, in which $Q_{-k} = \sum_{d\in D\setminus k} q_d$. The first-order condition (FOC) is $a - bQ_{-k} - 2bq_k - (1/t_k)q_k = 0$ or equivalently $bq_k = g_k(a - bQ)$. Summing up the FOC for all divisions in D

yields $Q = (a/b)(G/(1+G))$. Plug Q into the FOC to obtain $q_k = (a/b)(g_k/(1+G))$. Then, $\pi_k = (a^2 g_k(1+g_k))/(2b(1+G)^2)$.

Proof of Lemma 1. $n_i^L = \arg\max_{n_i} \pi_i(n_i, n_j)$ where $n_j = n^*(n_i)$. At $n_i = n_i^L$, $d\pi_i/dn_i = (\partial\pi_i/\partial n_i) + (\partial\pi_i/\partial n_j)(dn_j/dn_i) = 0$. But $\partial\pi_i/\partial n_j < 0$ while $dn_j/dn_i = n^{*\prime}(\cdot) > 0$, so $\partial\pi_i(n_i^L)/\partial n_i > 0$. Meanwhile, $n_i^M = \arg\max_{n_i} \pi_i(n_i, n_j)$ so $\partial\pi_i(n_i^M)/\partial n_i = 0$. Since $\partial\pi_i(n_i^L)/\partial n_i > \partial\pi_i(n_i^M)/\partial n_i$ and $\partial^2\pi_i/\partial n_i^2 < 0$, we conclude that $n_i^L < n_i^M$.

By symmetry, $n_j^L < n_j^M$. But $n_i^F = n^*(n_j^L)$ and $n_i^M = n^*(n_j^M)$. Because $n^{*\prime}(\cdot) > 0$, we have $n_i^F < n_i^M$. Now we must show that $n_i^L < n_i^F$. Suppose it is not (i.e., $n_i^L \geq n_i^F$). Then by symmetry, $n_j^L \geq n_j^F$,

$$
\begin{aligned}
n_i^F &= n^*\left(n_j^L\right) \\
&\geq n^*\left(n_j^F\right)\left(\text{because } n_j^L \geq n_j^F \text{ and } n^{*\prime}(\cdot) > 0\right) \\
&= \arg\max_{n_i} \pi_i\left(n_i, n_j^F\right) \\
&> \arg\max_{n_i} \pi_i\left(n_i, n^*\left(n_i\right)\right) \\
&\quad \left(\text{because } \frac{\partial\pi_i}{\partial n_j} < 0,\ n^{*\prime}(\cdot) > 0 \text{ and } n_j^F = n^*\left(n_i^L\right)\right) \\
&= n_i^L,
\end{aligned}
\tag{A.1}
$$

which is a clear contradiction.

For profits,

$$
\begin{aligned}
\pi_i^F &= \max_{n_i} \pi_i\left(n_i, n_j^L\right) \\
&> \max_{n_i} \pi_i\left(n_i, n_j^F\right) \\
&\quad \left(\text{by the envelope theorem, } \frac{\partial\pi_i}{\partial n_j} < 0, \text{ and } n_j^L < n_j^F\right) \\
&> \pi_i\left(n_i^L, n_j^F\right) \\
&= \pi_i^L \\
&= \max_{n_i} \pi_i\left(n_i, n^*\left(n_i\right)\right) \\
&> \pi_i\left(n_i^M, n^*\left(n_i^M\right)\right)\left(\text{because } n_i^L \neq n_i^M\right) \\
&= \pi_i^M.
\end{aligned}
\tag{A.2}
$$

□

Proof of Lemma 3. n^L is solved from $\beta(n^L) = 0$ or equivalently $\beta(n^L)/t_L^2 t_F^2 = 0$. When $t_i \to \infty$ and $t_j \to \infty$, $\beta(n^L)/t_L^2 t_F^2 = -2n^L + 2$, so $n^L = 1$. Then, $n^F = n^*(n^L) = 1 + (n^L t_L/(n^L + t_L)) = 2$. □

Proof of Proposition 4(i). Suppose that $s_i^L > s_i^F$. Then by symmetry, $s_j^L > s_j^F$. Because $s_i^L = n_i^L g_i^L/(n_i^L g_i^L + n_j^F g_j^F)$, we have

$$
\begin{aligned}
\pi_i^L &= \frac{n_i^L g_i^L\left(1 + g_i^L\right)}{\left(1 + n_i^L g_i^L + n_j^F g_j^F\right)^2} \\
&= s_i^L\left(1 + g_i^L\right)\frac{n_i^L g_i^L + n_j^F g_j^F}{\left(1 + n_i^L g_i^L + n_j^F g_j^F\right)^2}.
\end{aligned}
\tag{A.3}
$$

Similarly,

$$
\pi_j^F = s_j^F\left(1 + g_j^F\right)\frac{n_i^L g_i^L + n_j^F g_j^F}{\left(1 + n_i^L g_i^L + n_j^F g_j^F\right)^2}. \tag{A.4}
$$

Then, $\pi_i^L/\pi_j^F = s_i^L(1+g_i^L)/s_j^F(1+g_j^F)$. Similarly, $\pi_i^F/\pi_j^L = s_i^F(1+g_i^F)/s_j^L(1+g_j^L)$. Now, because $s_i^L > s_i^F$ and $s_j^L > s_j^F$, we have $s_i^L/s_j^F > s_i^F/s_j^L$. Because $n_i^L < n_i^F$ and $n_j^L < n_j^F$ (Lemma 1), we have $(1+g_i^L)/(1+g_j^F) = (1+(t_i/(n_i^L+t_i)))/(1+(t_j/(n_j^F+t_j))) > (1+(t_i/(n_i^F+t_i)))/(1+(t_j/(n_j^L+t_j))) = (1+g_i^F)/(1+g_j^L)$. As a result, we conclude that $\pi_i^L/\pi_j^F > \pi_i^F/\pi_j^L$ or equivalently $\pi_i^L/\pi_i^F > \pi_j^F/\pi_j^L$. By Lemma 1, $\pi_i^L < \pi_i^F$. Therefore, $\pi_j^F/\pi_j^L < \pi_i^L/\pi_i^F < 1$ or $\pi_j^F < \pi_j^L$. But this contradicts Lemma 1. □

Proof of Lemma 5. n^L is implicitly defined by $\beta(n^L) = 0$. Because $\beta(1) = t_F(1+2t_L)^2 \geq 0$ and $\beta(\infty) < 0$, $\beta(n^L) = 0$ has at least one root on $[1, \infty)$. Furthermore, the root is greater than 1 if and only if $t_F > 0$. Now it must be shown that this root is unique on $(0, \infty)$.

Suppose the contrary. Then, because $\beta(n^L)$ is cubic in n^L with $\beta(0) > 0$ and $\beta(\infty) < 0$, $\beta(n^L) = 0$ must have three roots on $(0, \infty)$. Further, it must be true that $\beta'(0) < 0$ and $\beta'(n^L) > 0$ at one of the roots.

Now, $\beta'(0) = t_L(6t_F + t_L + 2 + 2t_L t_F + 4t_F^2 - 2t_L t_F^2)$ and $[\beta'(n^L)]n^L - \beta(n^L) = 2\beta_3(n^L)^3 + \beta_2(n^L)^2 - \beta_0$, where $\beta_3 = -[(1+t_L+t_F)^2 + t_F^2] < 0$, $\beta_2 = [(1+t_F)(1+2t_F-t_L^2) - 4t_L t_F^2]$, and $\beta_0 = t_L^2(1+t_F)(1+2t_F) > 0$. It is straightforward to verify that $\beta'(0)/t_L > \beta_2$. Then, if $\beta_2 > 0$, it must be true that $\beta'(0) > 0$. If $\beta_2 < 0$, then $[\beta'(n^L)]n^L - \beta(n^L) < 0$, meaning that at any root (so $\beta(n^L) = 0$), $\beta'(n^L) < 0$. This is a contradiction because either case will violate the requirement that $\beta'(0) < 0$ and $\beta'(n^L) > 0$ at one of the roots. □

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Endnotes

1. This paper emphasizes the competition effects of divestiture—the intensified competition between divested units gives them a competitive advantage over rival firms. Real-life examples such as the Ford-Volvo divestiture inevitably involve multiple reasons including cost effects (diseconomies of scope) and product differentiation. But all divestitures, regardless of and on top of firm-specific and market-specific details and complications, invariably involve the competition effect, which is the focus of this study.
2. Divestiture is generally defined as the sale of a subsidiary, a division or a minority share to a new owner. In this paper, divestiture means breaking a firm up into autonomous units that compete independently in the product market. A common explanation for divestiture is diseconomies of scope associated with managing multiple product lines. In large organizations it becomes more and more difficult to manage complex tasks, transmit information, and motivate employees. When the costs of producing multiple products outweigh its benefits, divestiture becomes optimal. Bresnahan et al. [20] have recently suggested that diseconomies of scope may arise from the need for multiple products to share some common assets such as a credit rating, brand recognition, or the reputation of the management team. Since the diseconomies of scope motivation are straightforward and well understood, this paper abstracts away from this force by assuming away any cost advantage associated with divestiture. Instead, the driving force is assumed to arise through competition effects.
3. Lewis [21] and Schwartz and Thompson [22] have shown how setting up competing divisions may deter entry. Tan and Yuan [23] provided an alternative theory of divestitures: competing conglomerates may divest complementary product lines in order to mitigate product market competition.
4. Put another way, firms will divest more when their products are closer substitutes. Since firms hurt one another through their divestitures, a stronger incentive to divest would induce firms to seek ways to constrain their divestitures. Qiu and Zhou [11] showed that if mergers and divestitures are both possible in an industry, firms will avoid divestitures by merging with each other when their products are close substitutes. In that case, a moderate degree of product differentiation will be conducive to divestitures, while very strong or very weak differentiation will be conducive to mergers.
5. Product differentiation may thus explain divestitures observed in real life, such as the one cleaving Ford and Volvo. The two papers also explored other issues related to divestitures when products are differentiated. Ziss [3] showed that product differentiation will lead to a number of comparative statics results that are different from the ones obtained in previous studies assuming homogeneous products, while Yuan [4] demonstrated that entry will induce vigorous competition when incumbent firms have the option of setting up autonomous divisions, so divestiture can serve as a natural barrier to entry.
6. The same logic is also behind product proliferation [25]. Britain's Tesco stocks 91 different shampoos, 93 varieties of toothpaste, and 115 of household cleaner, many of which are produced by the same company. For example, Tropicana offers more than 20 different freshly pulped juices ("The tyranny of choice," *The Economist*, 12/16/2010).
7. A way out of the merger paradox is to alter or weaken the responsiveness: firms may compete on price rather than quantity [26]; marginal costs may be increasing [16]; or products may be differentiated [24]. Alternatively, the merger paradox disappears if mergers generate substantial benefits such as information pooling [24] or more efficient production in the face of cost uncertainty [27].
8. It is natural to assume that all firms can divest, so the endogenization is about the order of moves. A merger, by contrast, is inherently exogenous if the identity of the merging firms is exogenously determined, so endogenization can be in terms of many dimensions. If a firm can choose whether to join a single merger or stay out, the merger incentive will be further weakened [28], so the merger paradox cannot be solved by endogenization. If multiple mergers are allowed, the merger incentive can be strengthened [29]. Qiu and Zhou have shown that mergers are strategic complements, and that sequential mergers expand the scope of equilibrium mergers. This study has found that divestitures are also strategic complements, and sequential divestitures limit the scope of equilibrium divestitures.
9. This can be seen in real life [12, p.863]: "After Volvo and Ford merged, Ford's luxury brand, Jaguar, continued to compete with Volvo. A similar pattern was observed when Daimler and Chrysler merged. When Kimberly Clark and Scott Paper merged, Kleenex, the leading brand of Kimberly Clark in the facial tissue market, remained in competition with Scottie, the leading brand of Scott Paper." Note that even though Ford and Volvo maintained some competition after their merger, the competition was most credible and pushed to the largest extent only after Volvo had been spun off and operated as a truly independent entity.
10. Both firms possess the same constant-return-to-scale technology $q = (tl)^{1/2}$, in which t is capital and l is labor. In the short run, when its capital is fixed at t_i, firm i's variable cost is $C_i \equiv \min_{l_i} w l_i$ subject to $q_i = (t_i l_i)^{1/2}$, in which w is the wage rate. The optimization leads to a cost of $(w/t_i)\, q_i^2$, and the formulation assumed in the model results when the wage rate $w = 1/2$.
11. Commitment may seem to be an issue here. Suppose that y divests after x does. What does then prevent x's divisions from further divestiture after y's divestiture? In fact this is a challenge faced by all studies of divestitures regardless of the order of moves. One way out is to assume costly divestitures, in particular, a fixed cost for

divestiture that is independent of the divesting firm's capital. A division will then be less likely to divest than its parent because the division is smaller and therefore may not generate enough profit to cover the fixed divestiture cost. Note that this explanation requires divisions to be smaller than their parents, which is true only in the formulation assumed in this model. Veendorp [30] has shown in an entry-deterrence model that it is indeed optimal for the parent firm to allow divisions freedom in their operations but not in investment, which presumably would include setting up their own subdivisions.

12. It must be emphasized that equal division is optimal only when the number of divisions is optimally chosen. It may happen that an unequal distribution between two divisions generates greater total profits than an equal distribution, but the firm's optimal choice is then not to divest; that is, it will not have two divisions in equilibrium.

13. The demand intercept, a, enters the equilibrium profits only through the common coefficient a^2 and therefore does not affect a firm's divestiture choice as long as divestiture is costless. As for the demand slope, b, in addition to being a coefficient of the profit function, it also appears in the expression for g_k and therefore will affect the divestiture incentives. However, since b is always associated with t_k in the same manner, its normalization is only a rescaling of the capital stock, which will be the focus of the discussion.

14. Not divesting is a choice if $n_i = 1$. Later it will be shown that $n_i \geq 1$ in the equilibrium.

15. In this model divestitures are costless, so a firm's optimal divestiture depends only on the two firms' relative market shares. This leads to strategic complementarity. If divestitures were costly the strategic interdependence would be more complicated. Because a firm's profits are reduced by its rival's divestiture, if the rival divests more, the firm's divisions may not generate enough profit to cover the divestiture cost. In that case, the firm may respond by choosing to divest less; that is, divestitures may become strategic substitutes. That is why Baye et al. [2] found that a firm's optimal divestiture is inversely U-shaped in relation to its rivals' divestitures: divestitures are strategic complements when the rival divests little, but become strategic substitutes when the rival divests more extensively. Note that the nature of the strategic interdependence depends crucially on whether divestitures are costly. In all models where divestitures are assumed to be costless [1, 3–5], divestitures are strategic complements.

16. Although each individual division is smaller than the parent, all the divisions together "equal" the parent. Because the parent divides its capital equally among all its divisions, a divestiture does not impose any cost advantage or disadvantage. For any given total output, the divisions' joint production cost is exactly the same as the parent's was previously. Such a formulation facilitates focusing on the competition effects of divestitures without the complication of any cost effect due to, say, diseconomies of scope.

17. Both Ziss [3] and Yuan [4] assumed that products within a group are perfect substitutes. One wonders whether the within-firm substitutability can take other values or even be endogenized, and whether their conclusions still hold under alternative formulations about substitutability. As is shown in the appendix, firm k's profit is $\pi_k = (a - bQ_{-k} - bq_k)\,q_k - \left(q_k^2/2t_k\right)$, which can be rewritten as $\pi_k = \left[a - bQ_{-k} - \left(b + \left(1/2t_k\right)\right)q_k\right]q_k$. The latter expression can be viewed as the profit function of a firm with zero mc and differentiated products, as the own-elasticity captured by $b + \left(1/2t_k\right)$ is greater than the cross-elasticity captured by b. Firm asymmetry is then reflected in different own-elasticities for the two firms: $b + \left(1/2t_i\right) \neq b + \left(1/2t_j\right)$. Unlike the product differentiation formulation, the increasing mc approach does not require any additional ad hoc assumptions about the substitutability of newly created products when firms divest. By breaking up the parent's capital, a divestiture means that all divisions from the same firm produce symmetrically differentiated products. Divisions from firm i all have the same own-elasticity captured by $b + 1/\left(2t_i/n_i\right)$, which is greater than the cross-elasticity captured by b. Substitutability across firms remains asymmetric: $b + 1/\left(2t_i/n_i\right) \neq b + 1/\left(2t_j/n_j\right)$. Finally, demand for a division's product becomes more elastic than the demand for the parent's: $b + 1/\left(2t_i/n_i\right) > b + \left(1/2t_i\right)$.

18. Firm k's profit is $\pi_k = (a - bQ_{-k} - bq_k)\,q_k - \left(q_k^2/2t_k\right)$ when products are homogeneous. If products are differentiated, the profit becomes $\pi_k = (a - dQ_{-k} - bq_k)q_k - \left(q_k^2/2t_k\right)$ with $b \geq d > 0$, and $z = d/b$ represents the degree of product substitutability (using the notations of Yuan [4]). This profit expression can be rewritten as $\pi_k = \left[a - dQ_{-k} - \left(b + \left(1/2t_k\right)\right)q_k\right]q_k$, which is as if marginal costs were zero (and therefore constant), while the degree of product substitutability drops to $d/\left(b + \left(1/2t_k\right)\right)$. Product differentiation and increasing mc can therefore be regarded as two special cases of the more general formulation: $t_k = \infty$ if products are differentiated but $mc = 0$, while $d = b$ if products are homogeneous but mc is increasing. Given increasing mc, introducing product differentiation will change only the degree of substitutability, but the qualitative results of the model remain valid. In real life, products are mostly differentiated (e.g., as are Volvo versus Ford brands), and one can reasonably argue that marginal costs are increasing in the relevant range. Therefore, both elements may play a role in constraining the extent of divestiture in real life. In terms of modeling, since the role of product differentiation is well understood through the work of Ziss [3] and Yuan [4], this paper focused on two new features: endogenized order of divestitures and increasing marginal costs.

19. The result will not change when t_i and t_j differ and they change independently.

20. These graphs were generated using Maple 16 software. Proposition 4(i) is proved analytically in the appendix, and the validity of (ii) can be demonstrated by numerical calculation.
21. The two firms differ in both their capital stocks and their roles in sequential divestiture. To isolate the effects of the two asymmetries, it will be useful to look at simultaneous divestiture even though it is off the equilibrium path. It can be shown that when the two firms divest simultaneously, (i) the larger firm divests into fewer divisions, which would enlarge the size discrepancy between the two firms' divisions; (ii) the smaller firm's market share is reduced and, as a result, firm asymmetry is amplified; and (iii) the smaller firm is hurt more than the larger one.
22. The flip side of this result is that a merger between two small firms may improve social welfare [18].

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